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The onset of convection in a binary fluid saturated anisotropic porous layer

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ABSTRACT

The double diffusive convection in a horizontal fluid saturated anisotropic porous layer, which is heated and salted from below, is studied analytically. The generalized Darcy model is employed for the momentum equation. The critical Rayleigh number, wavenumber for stationary and oscillatory modes and frequency of oscillations are obtained analytically using linear theory. The effect of anisotropy parameters, solute Rayleigh number, Lewis number and Prandtl number on the stationary, oscillatory and finite amplitude convection is shown graphically. The transient behavior of the Nusselt and Sherwood numbers is studied, by solving numerically a fifth order Lorenz type system using Runge–Kutta method. Some of the convection systems previously reported in the literature are shown to be special cases of the system presented in this study.

1. Introduction

The problem of convection induced by temperature and concentration gradients or by concentration gradients of two species, known as double diffusive convection, has attracted considerable interest in the last two decades. If the gradients of two stratifying agencies such has heat and salt having different diffusivities are simultaneously present in a fluid layer, many interesting convective phenomena can occur that are not possible in a single component fluid. An excellent review of studies related to double diffusive convection has been reported by Turner [1–3], Huppert and Turner [4], Platten and Legros [5]. The double diffusive convection in porous medium has also become increasingly important in recent years because of its many applications in geophysics, particularly in saline geothermal fields.

The first linear stability analysis of thermohaline convection in a porous medium was performed by Nield [6]. He found the criteria for the existence of steady and oscillatory thermohaline convection. Finite amplitude convection in a two-component fluid saturated porous layer has been studied by Rudraiah et al. [7]. It is found that a vertical solute gradient sets up overstable motions. Rudraiah and Vortmeyer [8] have studied the onset of triple diffusive convection in a porous medium. They reported that the overstable and salt finger modes found to be simultaneously unstable when the density gradients due to the components are of the same sign and the effect of permeability is to suppress the convection range. The effect of porous medium coarseness on the onset of double diffusive convection was studied by Poulikakos [9]. The critical conditions for the onset of convection are documented. The problem of double diffusive convection in a fluid saturated porous layer was later on investigated by many authors [10–20]. Extensive reviews on this subject can be found in the book by Nield and Bejan [21].

Most of the studies have usually been concerned with homogeneous isotropic porous structures. In a porous medium, due to the structure of the solid material in which the fluid flows, there can be a pronounced anisotropy in such parameters as permeability or thermal diffusivity. The novelties introduced by anisotropy have only recently been studied. The geological and pedological processes rarely form isotropic medium as is usually assumed in transport studies. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Process such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous medium. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes.

Thermal convection in anisotropic porous media has attracted researchers only in the last three decades, despite its great relevance in engineering applications. There are few investigations available on the thermal convection in a fluid saturated anisotropic porous layer heated from below [22–33]. Castinel and Combarnous

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[22] have studied the Rayleigh–Benard convection in an anisotropic porous layer both, experimentally and theoretically. Epherre [23] extended their stability analysis to a porous medium with anisotropy in thermal diffusivity also. A theoretical analysis of non-linear thermal convection in an anisotropic porous medium is performed by Kvernvold and Tyvand [24]. Nilsen and Storesletten [25] have studied the problem of natural convection in both isotropic and anisotropic porous channels. Tyvand and Storesletten [26] investigated the problem concerning the onset of convection in an anisotropic porous layer in which the principal axes were obliquely oriented to the gravity vector. Storesletten [27] made a similar analysis for an anisotropic thermal diffusivity with axis non-orthogonal to the layer; the findings here are significantly different from the orthogonal case. Qin and Kaloni [28] develop a non-linear energy stability analysis for thermal convection in an anisotropic porous layer, using Brinkman equation when the permeability components are different in all three principal directions. Straughan and Walker [29] deal with a porous medium for which the permeability is transversely isotropic with respect to an inclined axis and allows for a non-Boussinesq density in the buoyancy force. They found that the nature of bifurcation into convection is very different from the Boussinesq situation and is always via an oscillatory instability. Payne et al. [30] studied penetrative convection in a porous medium with anisotropic permeability. They reported that the onset of convection may be via Hopf bifurcation rather than stationary convection. Penetrative convection in a horizontally isotropic porous layer with internal heat sink model and alternatively, a quadratic density temperature law is investigated by Carr and Putter [31] using non-linear energy analysis. The review of research on convective flow through anisotropic porous medium has been well documented by McKibbin [34,35] and Storesletten [36,37].

The study on convection in an anisotropic porous layer saturated with two-component fluid is not available except the work by Tyvand [38]. Therefore the aim of the present paper is to study the linear and non-linear double diffusive convection in an anisotropic porous layer.

2. Mathematical formulation

We consider an infinite horizontal fluid saturated porous layer confined between the planes $z = 0$ and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. A uniform adverse temperature gradient $\Delta T = (T_l - T_u)$ and a stabilizing concentration gradient $\Delta S = (S_l - S_u)$ where $T_l > T_u$ and $S_l > S_u$ are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The porous medium is assumed to possess horizontal isotropy in both thermal and mechanical properties. The Darcy model that includes the time derivative term is employed for the momentum equation. With these assumptions the basic governing equations are

$$\nabla\cdot\mathbf{q} = 0, \tag{2.1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t} + \nabla p + \mu\mathbf{K}\cdot\mathbf{q} - \rho\mathbf{g} = 0, \tag{2.2}$$

$$\gamma\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \nabla\cdot(\mathbf{D}\cdot\nabla T), \tag{2.3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\mathbf{q}\cdot\nabla)S = \kappa_S\nabla^2 S, \tag{2.4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)], \tag{2.5}$$

where $\mathbf{q} = (u, v, w)$ is the velocity, p the pressure, ε represents the porosity, $\mathbf{K} = K_x^{-1}(\mathbf{ii} + \mathbf{jj}) + K_z^{-1}(\mathbf{kk})$ the inverse of the anisotropic permeability tensor, T, the temperature, S, the concentration, $\mathbf{D} = D_x(\mathbf{ii} + \mathbf{jj}) + D_z(\mathbf{kk})$ the anisotropic heat diffusion tensor. ρ, μ, β_T and β_S denote the density, viscosity, thermal and solute expansion coefficients respectively, and κ_S is the mass diffusivity. It is hereby stated that permeability and heat diffusion are most strongly anisotropic than solute diffusivity. Therefore, we ignore the solute anisotropy. Unfortunately, we have no experimental support for this because measurement of anisotropic diffusivity is lacking. Further $\gamma = (\rho c)_m/(\rho c_p)_f$, $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon(\rho c_p)_f$, c_p is the specific heat of the fluid, at constant pressure, c is the specific heat of the solid, the subscripts f, s and m denotes fluid, solid and porous medium values, respectively. For simplicity, we assume the ratio of specific heat of fluid and solid phase to be unity.

2.1. Basic state

The basic state of the fluid is assumed to be quiescent and is given by,

$$\mathbf{q}_b = (0,0,0), \quad p = p_b(z), \quad T = T_b(z), \quad S = S_b(z), \quad \rho = \rho_b(z). \tag{2.6}$$

Using these into Eqs. (2.1)–(2.5) one can obtain

$$\frac{dp_b}{dz} = -\rho_b g, \frac{d^2T_b}{dz^2} = 0, \frac{d^2S_b}{dz^2} = 0, \quad \rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)]. \tag{2.7}$$

Then the conduction state temperature and concentration are given by

$$T_b = -\frac{\Delta T}{d}z + T_l, \quad S_b = -\frac{\Delta S}{d}z + S_l. \tag{2.8}$$

2.2. Perturbed state

On the basic state we superpose perturbations in the form

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}', T = T_b(z) + T', S = S_b(z) + S', p = p_b(z) + p', \quad \rho = \rho_b(z) + \rho', \tag{2.9}$$

where primes indicate perturbations. Introducing (2.9) into Eqs. (2.1)–(2.5) and using basic state solutions, we obtain the equations governing the perturbations in the form,

$$\nabla\cdot\mathbf{q}' = 0, \tag{2.10}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{q}'}{\partial t} + \nabla p' + \mu\mathbf{K}\cdot\mathbf{q}' - \left(\beta_S S' - \beta_T T'_f\right)\mathbf{g} = 0, \tag{2.11}$$

$$\gamma\frac{\partial T'}{\partial t} + (\mathbf{q}'\cdot\nabla)T' + w'\frac{\partial T_b}{\partial z} = \nabla\cdot(\mathbf{D}\cdot\nabla T'), \tag{2.12}$$

$$\varepsilon\frac{\partial S'}{\partial t} + (\mathbf{q}'\cdot\nabla)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S\nabla^2 S', \tag{2.13}$$

$$\rho' = -\rho_0\left[\beta_T T' - \beta_S S'\right]. \tag{2.14}$$

By operating curl twice on Eq. (2.11) we eliminate p' from it, and then render the resulting equation and the Eqs. (2.12)–(2.14) dimensionless using the following transformations

$$(x',y',z') = (x^*,y^*,z^*)d,\ t = t^*\left(\gamma d^2/D_z\right),\ (u',v',w') = (D_z/d)\left(u^*,v^*,w^*\right),\quad T' = (\Delta T)T^*,\ S' = (\Delta S)S^*, \tag{2.15}$$

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left[\frac{1}{\gamma Pr_D}\frac{\partial}{\partial t}\nabla^2 + \left(\nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]w - Ra_T\nabla_h^2 T + Ra_S\nabla_h^2 S = 0, \tag{2.16}$$

$$\left[\frac{\partial}{\partial t} - \left(\eta\nabla_h^2 + \frac{\partial^2}{\partial z^2}\right) + \mathbf{q}\cdot\nabla\right]T - w = 0, \tag{2.17}$$

$$\left[\varepsilon_n\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2 + \mathbf{q}\cdot\nabla\right]S - w = 0, \tag{2.18}$$

where $Pr_D = \varepsilon\nu d^2/K_z D_z$, the Darcy–Prandtl number, $Ra_T = \beta_T g\Delta T d K_z/\nu D_z$, the Darcy–Rayleigh number, $Ra_s = \beta_S g\Delta S d K_z/\nu D_z$, the solute Rayleigh number, $Le = D_z/\kappa_S$, the Lewis number, $\xi = K_x/K_z$, mechanical anisotropy parameter, $\eta = D_x/D_z$, thermal anisotropy parameter and $\varepsilon_n = \varepsilon/\gamma$ is the normalized porosity. To restrict the parameter space to the minimum, we set $\varepsilon = \gamma = 1$.

Equations (2.16)–(2.18) are to be solved for impermeable, isothermal and isohaline boundaries. Hence the boundary conditions for the perturbation variables are given by

$$w = T = S = 0 \quad \text{at } z = 0, 1. \tag{2.19}$$

3. Linear stability theory

In this section we predict the thresholds of both marginal and oscillatory convections using linear theory. The eigenvalue problem defined by Eqs. (2.16)–(2.18) subject to the boundary conditions (2.19) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} exp[i(lx + my) + \sigma t], \tag{3.1}$$

where l, m are horizontal wavenumbers and σ is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (3.1) into the linearized version of Eqs. (2.16)–(2.18) we obtain

$$\left[\frac{\sigma}{Pr_D}(D^2 - a^2) + \left(\frac{D^2}{\xi} - a^2\right)\right]W + a^2 Ra_T\Theta - a^2 Ra_S\Phi = 0, \tag{3.2}$$

$$\left[\sigma - (D^2 - \eta a^2)\right]\Theta - W = 0, \tag{3.3}$$

$$\left[\sigma - \frac{1}{Le}(D^2 - a^2)\right]\Phi - W = 0, \tag{3.4}$$

where $D = d/dz$ and $a^2 = l^2 + m^2$. The boundary conditions (2.19) now read

$$W = \Theta = \Phi = 0 \quad \text{at } z = 0, 1. \tag{3.5}$$

We assume the solutions of Eqs. (3.2)–(3.4) satisfying the boundary conditions (3.5) in the from

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin n\pi z, \quad (n = 1, 2, 3, \ldots). \tag{3.6}$$

The most unstable mode corresponds to $n = 1$ (fundamental mode). Therefore, substituting Eq. (3.6) with $n = 1$ into Eqs. (3.2)–(3.4), we obtain a matrix equation

$$\begin{pmatrix} \delta^2\sigma Pr_D^{-1} + \delta_1^2 & -a^2 Ra_T & a^2 Ra_S \\ -1 & \sigma + \delta_2^2 & 0 \\ -1 & 0 & \sigma + \delta^2 Le^{-1} \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{3.7}$$

where $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \pi^2\xi^{-1} + a^2$ and $\delta_2^2 = \pi^2 + \eta a^2$. The condition of nontrivial solution of above system of homogeneous linear equations (3.7) yields the expression for thermal Rayleigh number in the form

$$Ra_T = \frac{\left(\sigma + \delta_2^2\right)}{a^2}\left(\frac{\sigma\delta^2}{\mathrm{Pr}_D} + \delta_1^2 + \frac{a^2 Ra_S}{\sigma + \delta^2 Le^{-1}}\right). \tag{3.8}$$

The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\sigma_i$. The system with $\sigma_r < 0$ is always stable, while for $\sigma_r > 0$ it will become unstable. For neutral stability state $\sigma_r = 0$.

3.1. Stationary mode

For the validity of principle of exchange of stabilities (i.e., steady case), we have $\sigma = 0$ (i.e., $\sigma_r = \sigma_i = 0$) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$Ra_T^{St} = \frac{1}{a^2}\left(a^2 + \frac{\pi^2}{\xi}\right)(\eta a^2 + \pi^2) + \left(\frac{\eta a^2 + \pi^2}{a^2 + \pi^2}\right)LeRa_S. \tag{3.9}$$

The minimum value of the Rayleigh number Ra_T^{St} occurs at the critical wavenumber $a = a_c^{St}$ where $a_c^{St} = \sqrt{x}$ satisfies the equation

$$\eta x^4 + 2\pi^2\eta x^3 + \pi^2\left(\pi^2(\eta - \xi^{-1}) + LeRa_S(\eta - 1)\right)x^2 - 2\pi^6\xi^{-1}x - \pi^8\xi^{-1} = 0. \tag{3.10}$$

It is important to note that the critical wavenumber a_c^{St} depends on the solute Rayleigh number apart from the Lewis number and anisotropic properties. This result is in contrast to the case of isotropic porous medium. For an isotropic porous media, that is when $\xi = \eta = 1$, Eq. (3.9) gives

$$Ra_T^{St} = \frac{1}{a^2}(a^2 + \pi^2)^2 + LeRa_S. \tag{3.11}$$

which is the classical result for a double diffusive convection in an isotropic porous media (see e.g. [21]). For single component fluid, $Ra_S = 0$, the expression for stationary Rayleigh number given by Eq. (3.9) reduces to

$$Ra_T^{St} = \frac{1}{a^2}\left(a^2 + \frac{\pi^2}{\xi}\right)(\eta a^2 + \pi^2), \tag{3.12}$$

which is the one obtained by Storesletten [36] for the case of single component fluid saturated anisotropic porous layer. Further, for an isotropic porous medium, $\xi = \eta = 1$, the above Eq. (3.12) reduces to the classical result

$$Ra_T^{St} = \frac{1}{a^2}\left(a^2+\pi^2\right)^2, \tag{3.13}$$

which has the critical value $Ra_c^{St} = 4\pi^2$ for $a_c^{St} = \pi^2$ obtained by Horton and Rogers [39] and Lapwood [40].

3.2. Oscillatory motion

We now set $\sigma = i\sigma_i$ in Eq. (3.8)and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\sigma_i\Delta_2. \tag{3.14}$$

Here the expressions for Δ_1 and Δ_2 are not presented for brevity. Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.14) it follows that either $\sigma_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\sigma_i \neq 0$, oscillatory onset).For oscillatory onset $\Delta_2 = 0(\sigma_i \neq 0)$ and this gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$\sigma^2 = \frac{a^2 Ra_S\left(\delta_2^2 - \delta^2 Le^{-1}\right)}{\left(\delta_1^2 + \delta^2\delta_2^2 Pr_D^{-1}\right)} - \left(\frac{\delta^2}{Le}\right)^2. \tag{3.15}$$

Now Eq. (3.14) with $\Delta_2 = 0$, gives

$$Ra^{Osc} = \frac{1}{a^2}\left(\delta_1^2\delta_2^2 - \frac{\sigma^2\delta^2}{Pr_D}\right) + \frac{Ra_S\left(\sigma^2 + \delta^2\delta_2^2 Le^{-1}\right)}{\sigma^2 + \left(\delta^2 Le^{-1}\right)^2}. \tag{3.16}$$

The analytical expression for oscillatory Rayleigh number given by Eq. (3.16) is minimized with respect to the wavenumber numerically, after substituting for σ^2 (>0) from Eq. (3.15), for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

4. Weak non-linear theory

In this section we consider the non-linear analysis using a truncated representation of Fourier series considering only two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigenfunctions describing qualitatively the convective flow, it can neither provide information about the values of the convection amplitudes, nor regarding the rate of heat and mass transfer. To obtain this additional information, we perform the non-linear analysis, which is useful to understand the physical mechanism with minimum amount of mathematics and is a step forward towards understanding the full non-linear problem.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of y. We introduce stream function ψ such that $u = \partial\psi/\partial z$, $w = -\partial\psi/\partial x$ into the Eq. (2.11), eliminate pressure and non-dimensionalize the resulting equation and Eqs. (2.12)–(2.13) using the transformations (2.15) to obtain

$$\frac{1}{\gamma Pr_D}\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{4.1}$$

$$\frac{\partial T}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)T - \frac{\partial(\psi,T)}{\partial(x,z)} + \frac{\partial\psi}{\partial x} = 0, \tag{4.2}$$

$$\varepsilon_n\frac{\partial S}{\partial t} - \frac{1}{Le}\nabla^2 S - \frac{\partial(\psi,S)}{\partial(x,z)} + \frac{\partial\psi}{\partial x} = 0. \tag{4.3}$$

As mentioned earlier, we set $\gamma = \varepsilon = 1$ for simplicity. For flows with $Ra_T > Ra_{Tc}$ the linear stability analysis is not valid one has to take into account the non-linear effects. The first effect of non-linearity is to distort the temperature and concentration fields through the interaction of ψ, T and also ψ, S. The distortion of these fields will corresponds to a change in the horizontal mean, i.e., a component of the form $sin(2\pi z)$ will be generated. Thus a minimal Fourier series which describes the finite amplitude free convection is given by,

$$\psi = A(t)\sin(ax)\sin(\pi z), \tag{4.4}$$

$$T = B(t)\cos(ax)\sin(\pi z) + C(t)\sin(2\pi z), \tag{4.5}$$

$$S = E(t)cos(ax)sin(\pi z) + F(t)sin(2\pi z), \tag{4.6}$$

where the amplitudes $A(t)$, $B(t)$, $C(t)$, $E(t)$ and $F(t)$ are to be determined from the dynamics of the system.

Substituting Eqs. (4.4)–(4.6) into Eqs. (4.1)–(4.3) and equating the coefficients of like terms we obtain the following non-linear autonomous system of differential equations

$$\frac{dA}{dt} = -\frac{Pr_D}{\delta^2}\left(\delta_1^2 A + aRa_T B - aRa_S E\right), \tag{4.7}$$

$$\frac{dB}{dt} = -\left(aA - \delta_2^2 B - \pi aAC\right), \tag{4.8}$$

$$\frac{dC}{dt} = -4\pi^2 C + \frac{\pi a}{2}AB, \tag{4.9}$$

$$\frac{dE}{dt} = -aA - \frac{\delta^2}{Le}E - \pi aAF, \tag{4.10}$$

$$\frac{dF}{dt} = -\frac{4\pi^2}{Le}F + \frac{\pi a}{2}AE. \tag{4.11}$$

The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below. The system of equations (4.7)–(4.11) is uniformly bounded in time

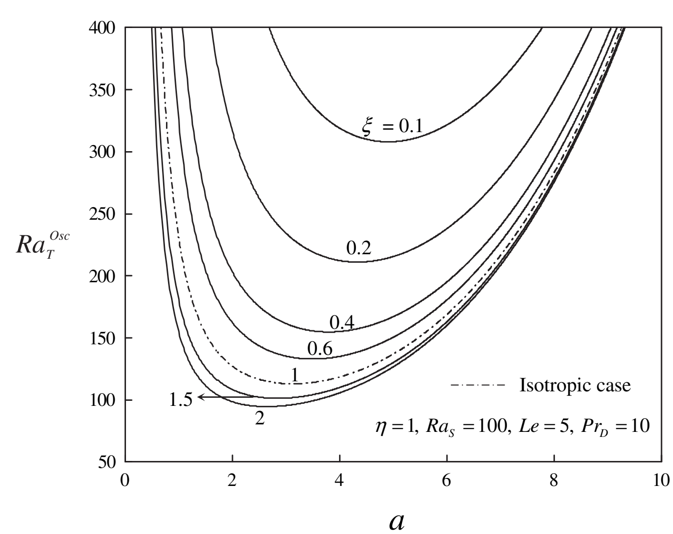

Fig. 1. Oscillatory neutral stability curves for different values of mechanical anisotropy parameter ξ.

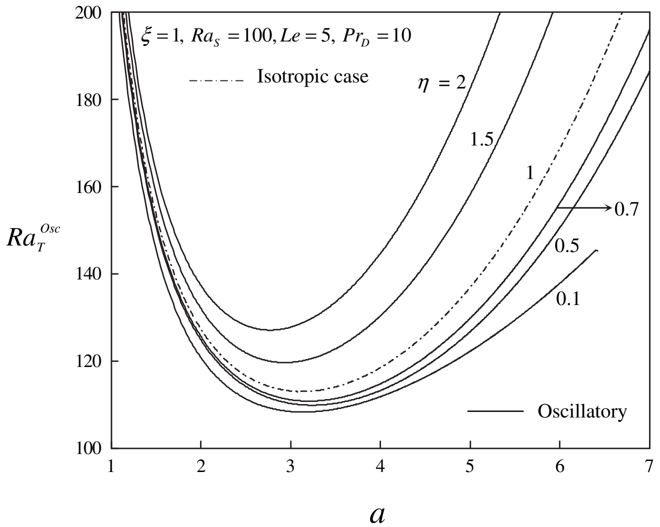

Fig. 2. Oscillatory neutral stability curves for different values of thermal anisotropy parameter η.

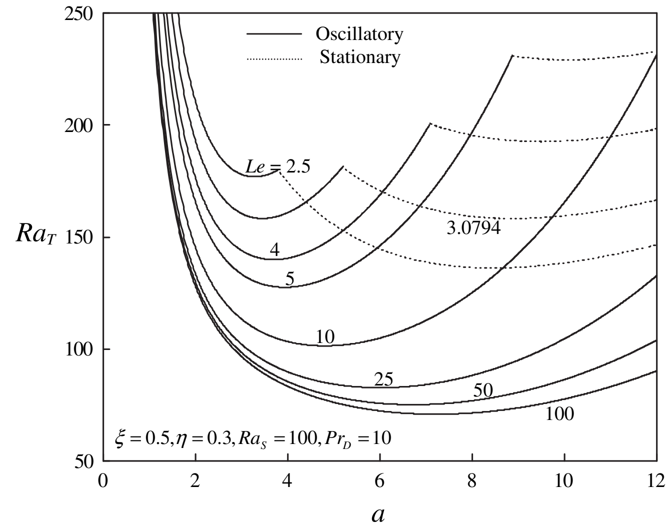

Fig. 4. Neutral stability curves for different values of Lewis number Le.

and possesses many properties of the full problem. Like the original equations (2.16)–(2.18), Eqs. (4.4)–(4.8) must be dissipative. Thus volume in the phase space must contract. In order to prove volume contraction, we must show that velocity field has a constant negative divergence. Indeed,

$$\frac{\partial}{\partial A}\left(\frac{dA}{dt}\right) + \frac{\partial}{\partial B}\left(\frac{dB}{dt}\right) + \frac{\partial}{\partial C}\left(\frac{dC}{dt}\right) + \frac{\partial}{\partial E}\left(\frac{dE}{dt}\right) + \frac{\partial}{\partial F}\left(\frac{dF}{dt}\right) = -\left[\frac{Pr_D\delta_1^2}{\delta^2} + \left(\delta_2^2 + 4\pi^2\right) + \frac{1}{Le}(\delta^2 + 4\pi^2)\right], \tag{4.12}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. From Eq. (4.12) we conclude that if a set of initial points in phase space occupies a region $V(0)$ at time $t = 0$, then after some time t, the end points of the corresponding trajectories will fill a volume

$$V(t) = V(0)\exp\left[-\left(\frac{Pr_D\delta_1^2}{\delta^2} + \left(\delta_2^2 + 4\pi^2\right) + \frac{1}{Le}(\delta^2 + 4\pi^2)\right)t\right]. \tag{4.13}$$

This expression indicates that the volume decreases exponentially with time. We can also infer that, the large Darcy–Prandtl number and very small Lewis number ($Le < 1$) tend to enhance dissipation.

Finally we note that the system of Eqs. (4.7)–(4.11) are invariant under the symmetry transformation $(A, B, C, E, F) \rightarrow (-A, -B, C, -E, -F)$.

4.1. Steady finite amplitude motions

From qualitative predictions we look into the possibility of an analytical solution. In the case of steady motions, Eqs. (4.1)–(4.3) can be solved in closed form. Setting the left hand sides of Eqs. (4.7)–(4.11) equal to zero, we get

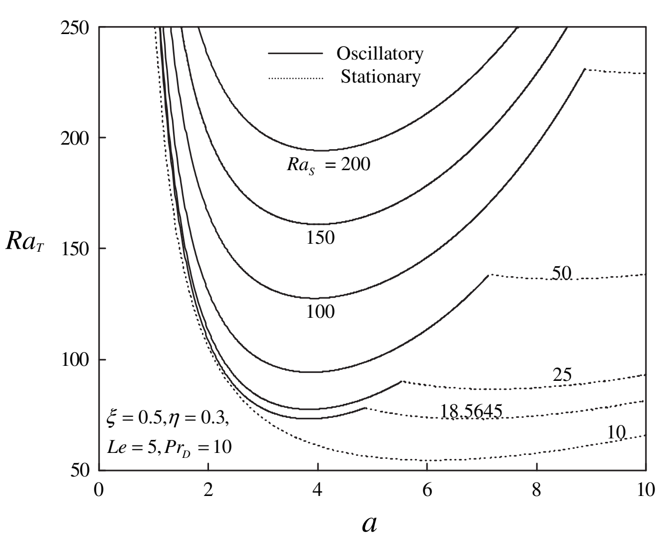

Fig. 3. Neutral stability curves for different values of solute Rayleigh number Ra_S.

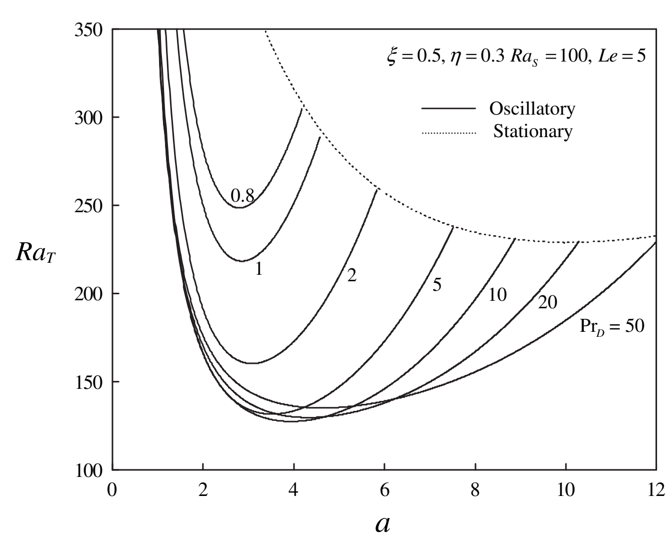

Fig. 5. Neutral stability curves for different values of Darcy–Prandtl number Pr_D.

Table 1
Critical values of solute Rayleigh number (for $Le = 5$) and Lewis number (for $Ra_S = 100$) for different values of ξ ($\eta = 0.3$, $Pr_D = 10$).

ξ	$(Ra_S)_c$	Le_c
0.1	62.71770	4.32300
0.2	35.68450	3.67140
0.4	21.54900	3.20105
0.5	18.56450	3.07942
0.6	16.55250	3.00481
1.0	12.38005	2.77600
2.0	9.02150	2.56571

$$\delta_1^2 A + aRa_T B_1 - aRa_S C = 0, \tag{4.14}$$

$$aA + \delta_2^2 B + \pi a A C = 0, \tag{4.15}$$

$$8\pi^2 C - \pi a A B = 0, \tag{4.16}$$

$$aA + \frac{\delta^2}{Le}E + \pi a A F = 0, \tag{4.17}$$

$$\frac{8\pi^2}{Le}F - \pi a A E = 0. \tag{4.18}$$

Writing B, C, E and F in terms of A, using Eqs. (4.15)–(4.18) and substituting these in Eq. (4.14), with $A^2/8 = x$ we get

$$A_1 x^2 + A_2 x + A_3 = 0, \tag{4.19}$$

where $A_1 = a^4\delta_1^2 Le^2$, $A_2 = a^2\delta_1^2(\delta^2 + \delta_2^2 Le^2) + a^4 Le(Ra_S - LeRa_T)$,

$A_3 = \delta^2\delta_1^2\delta_2^2 + a^2(\delta_2^2 LeRa_S - \delta^2 Ra_T)$.

The required root of Eq. (4.19) is,

$$x = \frac{1}{2A_1}\left(-A_2 + \left(A_2^2 - 4A_1A_3\right)^{1/2}\right). \tag{4.20}$$

When we let the radical in the above equation to vanish, we obtain the expression for finite amplitude Rayleigh number Ra^F, which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra^F = \frac{1}{2B_1}\left(-B_2 + (B_2 - 4B_1B_3)^{1/2}\right), \tag{4.21}$$

where $B_1 = a^4 Le^4$, $B_2 = 2a^2 Le^2\left(\delta_1^2\left(\delta^2 - \delta_2^2 Le^2\right) - a^2 LeRa_S\right)$,

$$B_3 = \left[a^2 LeRa_S + \delta_1^2\left(\delta^2 - \delta_2^2 Le^2\right)\right]^2.$$

The expression for the steady finite amplitude Rayleigh number given by Eq. (4.21) is evaluated for critical values and the results are discussed in Section 5.

Table 2
Critical values of solute Rayleigh number (for $Le = 5$) and Lewis number (for $Ra_S = 100$) for different values of η ($\xi = 0.5$, $Pr_D = 10$).

η	$(Ra_S)_c$	Le_c
0.1	503.29320	5.811715
0.2	32.41470	3.882450
0.3	18.56450	3.079420
0.5	11.41550	2.366970
0.7	8.93300	2.034500
1.0	7.21292	1.779120
2.0	5.30186	1.495250

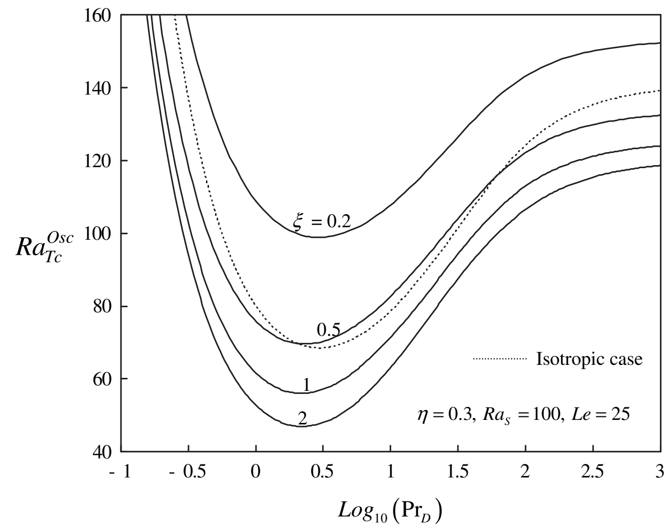

Fig. 6. Variation of oscillatory critical Rayleigh number with Darcy–Prandtl number Pr_D for different values of mechanical anisotropy parameter ξ.

4.2. Heat and mass transports

In the study of convection in fluids, the quantification of heat and mass transport is important. This is because the onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone.

If H and J are the rate of heat and mass transport per unit area respectively, then

$$H = -D_z\left\langle\frac{\partial T_{\mathrm{total}}}{\partial z}\right\rangle_{z=0}, \tag{4.22}$$

$$J = -\kappa_S\left\langle\frac{\partial S_{\mathrm{total}}}{\partial z}\right\rangle_{z=0}, \tag{4.23}$$

where the angular bracket corresponds to a horizontal average and

$$T_{\mathrm{total}} = T_0 - \Delta T\frac{z}{d} + T(x, z, t), \tag{4.24}$$

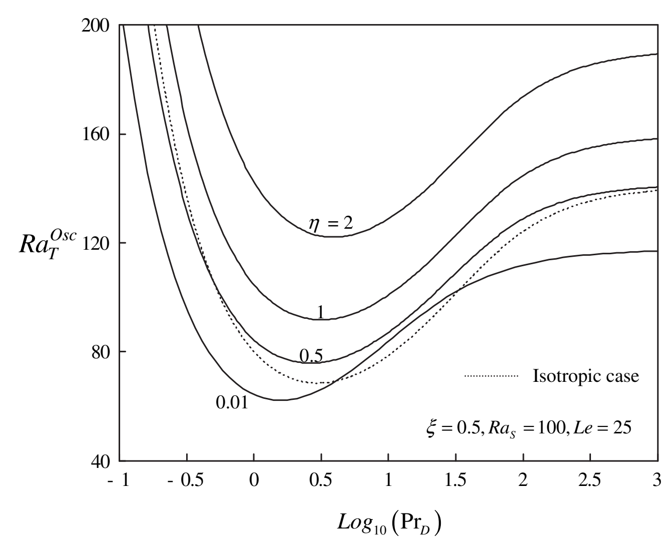

Fig. 7. Variation of oscillatory critical Rayleigh number with Darcy–Prandtl number Pr_D for different values of thermal anisotropy parameter η.

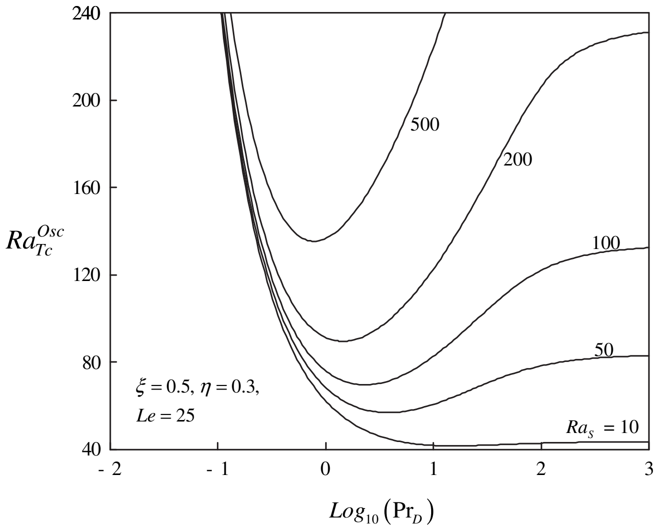

Fig. 8. Variation of oscillatory critical Rayleigh number with Darcy–Prandtl number Pr_D for different values of solute Rayleigh number Ra_S.

$$S_{\text{total}} = S_0 - \Delta S\frac{z}{d} + S(x, z, t). \tag{4.25}$$

Substituting Eqs. (4.8) and (4.9) in Eqs. (4.24) and (4.25) respectively and using the resultant equations in Eqs. (4.22) and (4.23), we get

$$H = \frac{D_z \Delta T}{d}(1 - 2\pi C), \tag{4.26}$$

$$J = \frac{\kappa_S \Delta S}{d}(1 - 2\pi F). \tag{4.27}$$

The Nusselt number and Sherwood number are defined by

$$Nu = \frac{H}{D_z \Delta T/d} = 1 - 2\pi C, \tag{4.28}$$

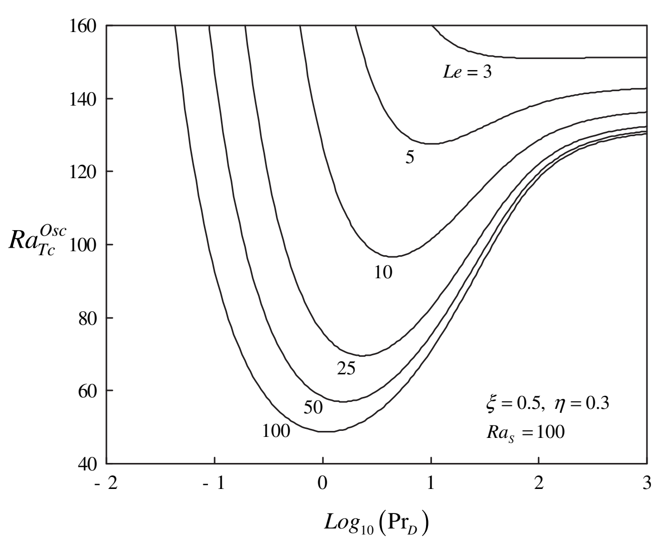

Fig. 9. Variation of oscillatory critical Rayleigh number with Darcy–Prandtl number Pr_D for different values of Lewis number Le.

$$Sh = \frac{J}{\kappa_S \Delta S/d} = (1 - 2\pi F). \tag{4.29}$$

Writing C and F in terms of A, using Eqs. (4.13)–(4.17), and substituting in Eqs. (4.28) and (4.29) respectively, we obtain

$$Nu = 1 + \frac{2x}{x + \delta_2^2/a^2}, \tag{4.30}$$

$$Sh = 1 + \frac{2x}{x + \delta^2/a^2 Le^2}. \tag{4.31}$$

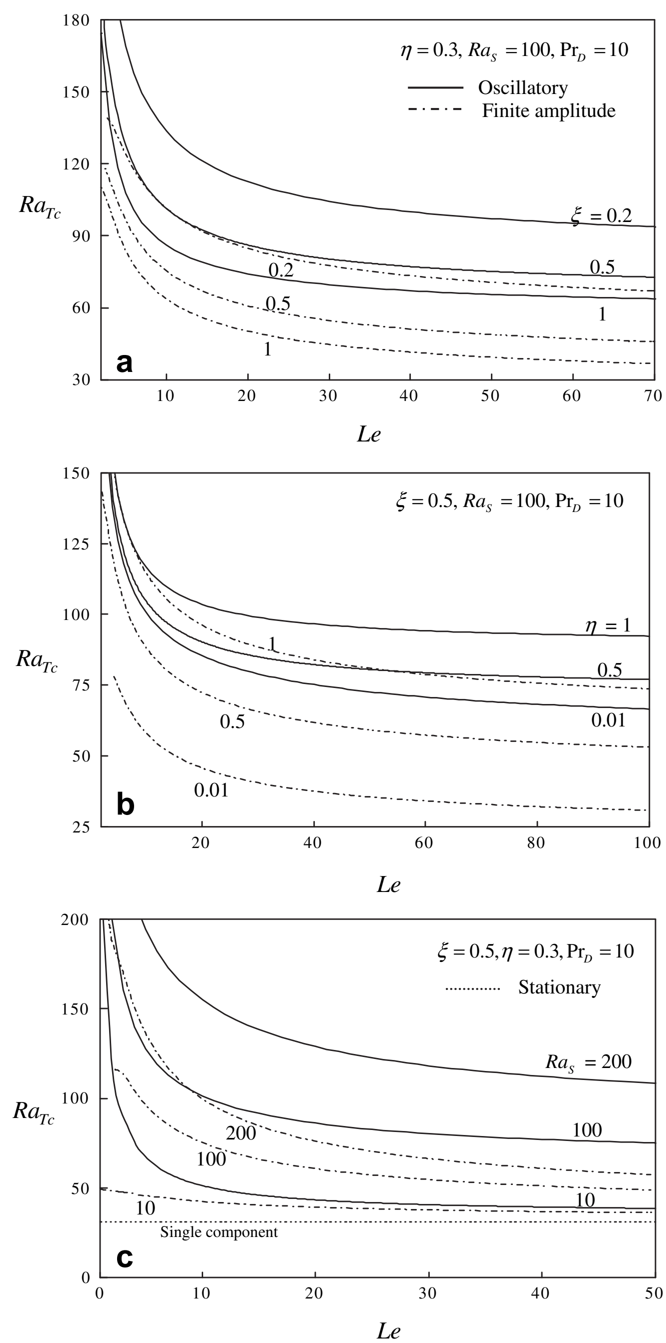

Fig. 10. Variation of critical Rayleigh number with Lewis number Le for different values of (a) ξ, (b) η and (c) Ra_S.

The second term on the right-hand side of Eqs. (4.30) and (4.31) represent the convective contribution to heat and mass transport respectively.

To know the transient behavior of Nusselt number and Sherwood number the autonomous system of Eqs. (4.7)–(4.11) have been solved numerically using the Runge–Kutta method with appropriate initial conditions for different values of Ra_c^F and the expressions for the Nusselt number and Sherwood number are computed.

5. Results and discussion

The onset of double diffusive convection in a two-component Boussinesq fluid saturated anisotropic porous layer, which is heated and salted from below, is investigated analytically using the linear and non-linear theories. In the linear stability theory the expressions for the stationary and oscillatory Rayleigh number are obtained analytically along with the expression for frequency of oscillation. The non-linear theory provides the quantification of heat and mass transports and also explains the possibility of the finite amplitude motions.

The neutral stability curves in the $Ra_T - a$ plane for various parameter values are shown in Figs. 1–5. We fixed the values for the parameters as $\xi = 0.5$, $\eta = 0.3$ $Ra_S = 100$, $Le = 5$ and $Pr_D = 10$ except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, Ra_{Tc}, below which the system is stable and unstable above. The points where the overstable solutions branch off from the stationary convection can be easily identified from these figures. Also we observe that for smaller values of the wavenumber each curve is a margin of the oscillatory instability and at some fixed a depending on the other parameters the overstability disappears and the curve forms the margin of stationary convection.

The effect of mechanical anisotropy parameter ξ for the fixed values of other parameters on the marginal stability curves is depicted in Fig. 1. It can be observed that an increase in ξ decreases the minimum of the Rayleigh number for oscillatory state, indicating that, the effect of increasing mechanical anisotropy parameter ξ is to advance the onset of oscillatory convection. The effect of mechanical anisotropy can be understood as fallows; let us keep the vertical permeability K_z fixed and then an increased horizontal permeability K_x reduces the critical Rayleigh number. This is due to the fact that an increased horizontal permeability enhances the fluid mobility in the vertical direction and hence convection sets in early. On the other hand keep horizontal permeability K_x fixed then an increased vertical permeability K_z, increases the critical Rayleigh number. Further, we find that the minimum of Rayleigh number shift towards the smaller values of the wavenumebr with increasing mechanical anisotropy parameter. This indicates that the cell width increases with increasing mechanical anisotropy parameter.

Fig. 2 indicates the effect of thermal anisotropy parameter η on the oscillatory neutral curves for the fixed values of ξ, Ra_S, Le and Pr_D. It is observed that critical value of Rayleigh number increases with η, indicating that the effect of thermal anisotropy parameter η is to inhibit the onset of oscillatory convection. Fig. 3 depicts the effect of solute Rayleigh number Ra_S on the neutral curves. We find that the effect of increasing Ra_S is to increase the critical value of the Rayleigh number and the corresponding wavenumber. Thus the solute Rayleigh number Ra_S has a stabilizing effect on the double

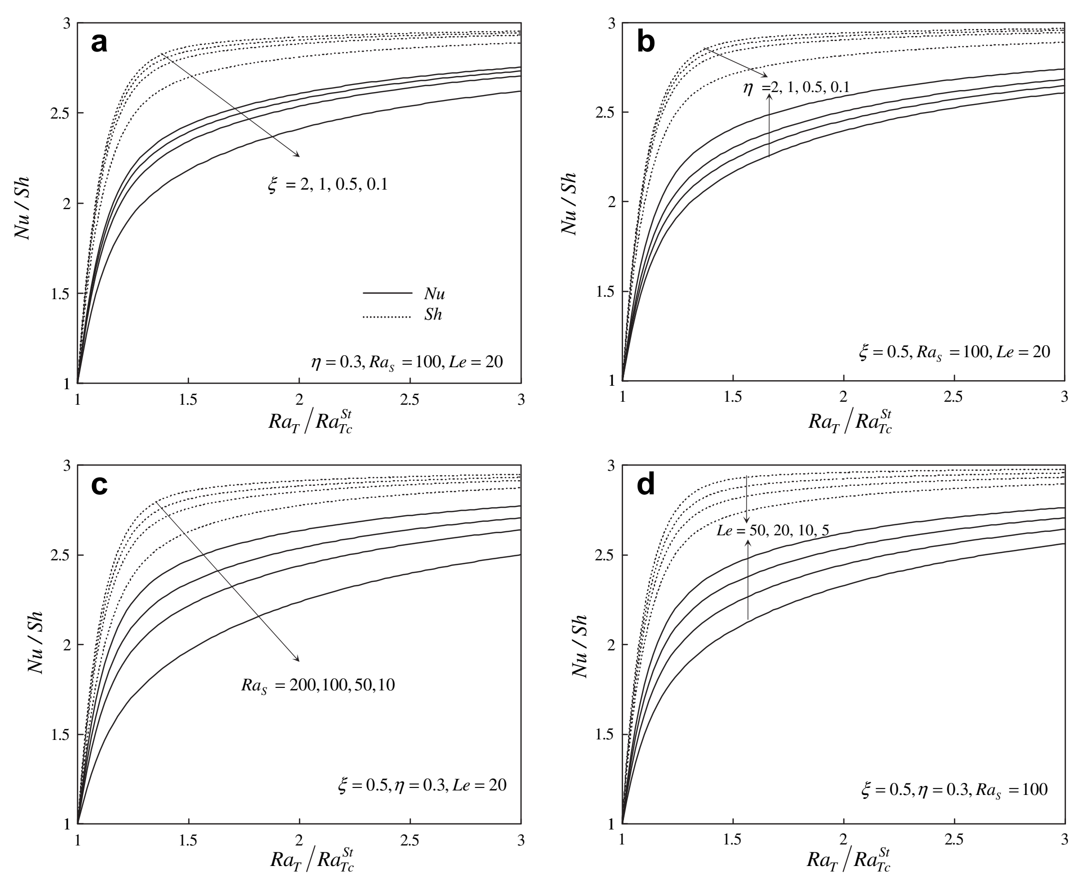

Fig. 11. Variation of Nusselt number and Sherwood number with critical Rayleigh number for different values of (a) ξ, (b) η, (c) Ra_S and (d) Le.

diffusive convection in porous medium. Fig. 3 is interesting and depicts a striking effect of the solute Rayleigh number. We find that for fixed values of all other parameters, there exists critical solute Rayleigh number $(Ra_S)_c$ such that for $Ra_S < (Ra_S)_c$ convection first sets in through stationary mode and for $Ra_S > (Ra_S)_c$ the convection mode switches to the oscillatory type. Further, when $Ra_S = (Ra_S)_c$, the stationary and oscillatory convection occur simultaneously with different critical wavenumbers. For fixed parameter values chosen for this figure, for example, it is found that $(Ra_S)_c = 18.5645$. Tables 1 and 2 show the critical values of the solute Rayleigh number at which the convection mode switches from stationary to the oscillatory, for different values of the mechanical and thermal anisotropy parameters.

In Fig. 4 the marginal stability curves for different values of Lewis number Le are drawn. It is observed that with the increase of Le the critical values of Rayleigh number and the corresponding wavenumber for the overstable mode decrease while those for stationary mode increase. Therefore, the effect of Le is to advance the onset of oscillatory convection where as its effect is to inhibit the stationary onset. Further it is important to note that there exists critical value of the Lewis number $(Le)_c$ at which the convection switches from stationary to the oscillatory. For example, $(Le)_c = 3.0794$ when the other parameter values are fixed as $\xi = 0.5$, $\eta = 0.3$, $Ra_S = 100$, $Pr_D = 10$(see Fig. 4). The critical values of the Lewis number at which the convection mode switches from stationary to the oscillatory, for different values of the

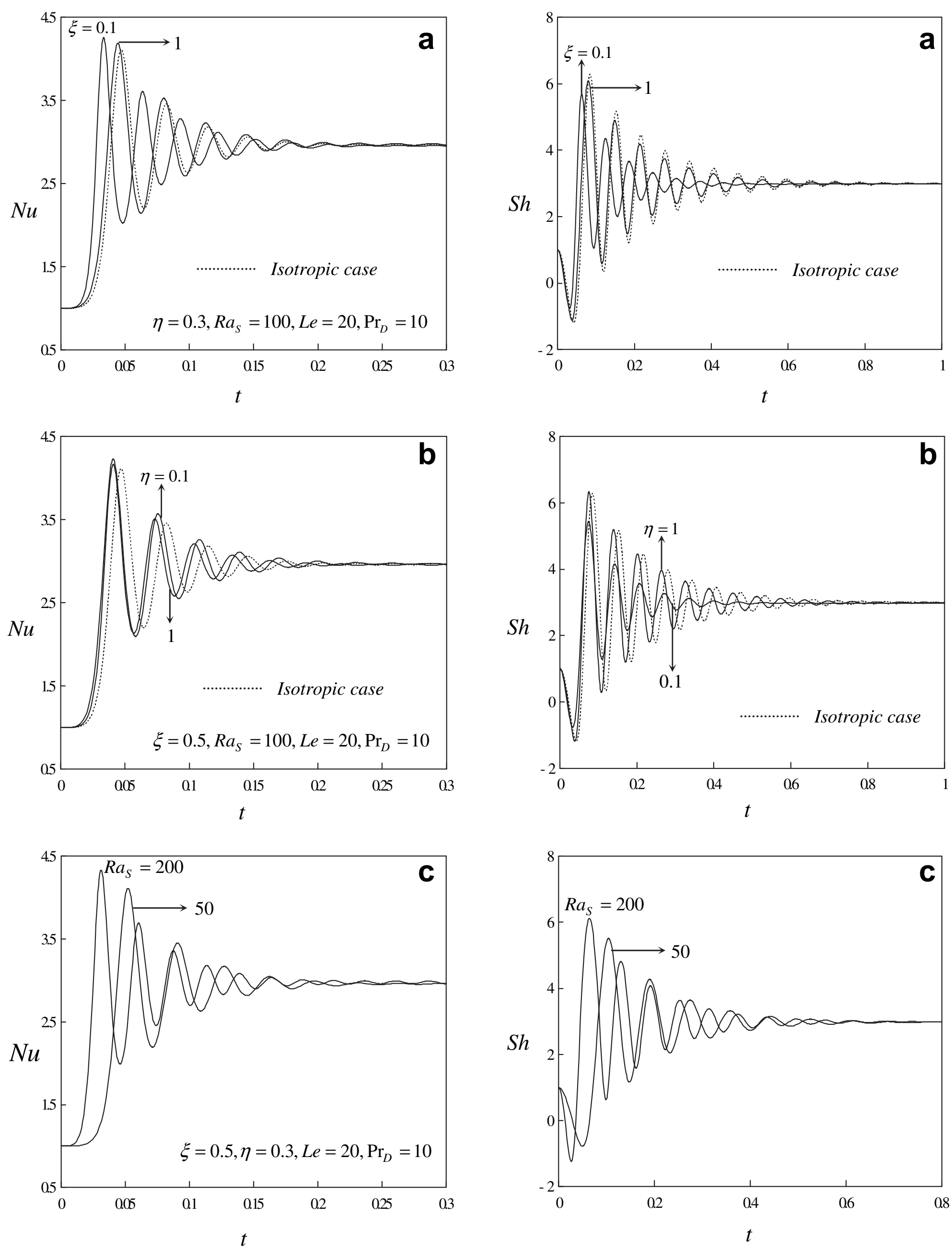

Fig. 12. Variation of Nusselt number and Sherwood number with time for different values of (a) ξ, (b) η (c) Ra_S. (d) Le, and (e) Pr_D.

mechanical and thermal anisotropy parameters is given in Tables 1 and 2.

The neutral stability curves for different values of Darcy–Prandtl number Pr_D are presented in Fig. 5. The point where the overstable solution bifurcates into the stationary solution is observed to be shifted towards a higher value of a with the increasing Pr_D. From this figure it is evident that for small and moderate values of Pr_D the critical value of oscillatory Rayleigh number decreases with the increase of Pr_D, however this trend is reversed for large values of Pr_D.

The detailed behavior of oscillatory critical Rayleigh number with respect to the Darcy–Prandtl number is analysed in the $Ra_{Tc}^{Osc} - Pr_D$ plane through Figs. 6–9. From these figures it is evident that there is a critical value of Pr_D, say Pr_D^* (depending on other parameters) such that for the values $Pr_D < Pr_D^*$ the oscillatory critical Rayleigh number decreases with the increase in Pr_D, while for the values of $Pr_D > Pr_D^*$ the Ra_{Tc}^{Osc} increases. Therefore, for the oscillatory mode a strong destabilizing effect is observed when $Pr_D < Pr_D^*$, however when Pr_D is increased beyond Pr_D^* this effect is reversed and hence the system is stabilized. The stabilizing effect of Pr_D becomes less significant when $Pr_D >> Pr_D^*$.

In Fig. 6 the variation of Ra_{Tc}^{Osc} with Pr_D for different values of mechanical anisotropy parameter ξ is shown for the fixed values of other parameters. It is important to note that Ra_{Tc}^{Osc} decreases with the increase of ξ. The critical curves corresponding to large values of ξ ($\xi > 0.6$, for $\eta = 0.3$, $Ra_S = 100$, $Le = 25$) always lie below the critical curves corresponding to isotropic case ($\xi = \eta = 1$). Thus the effect of mechanical anisotropy of the porous layer is to advance the onset of oscillatory convection as compared to the isotropic porous layer for $\xi > 0.6$ while reverse holds for $\xi \leq 0.5$ when other parameters are held as above.

Fig. 7 indicates the variation of Ra_{Tc}^{Osc} with Pr_D for different values of thermal anisotropy parameter η. It is observed that Ra_{Tc}^{Osc} increases with the increase of η. Thus the effect of thermal anisotropy parameter is to inhibit the onset of oscillatory convection as compared to the isotropic porous layer. It is important to note that the value of $Pr_D = Pr_D^*$, at which the system becomes most unstable increases with η.

The effect of solute Rayleigh number Ra_S and Lewis number Le on the onset criteria is shown in Figs. 8 and 9 respectively. We observe from these figures that the effect of Ra_S is analogous to that of η and effect of Le is similar to of ξ. That is the solute Rayleigh number makes the system more stable while the Lewis number is responsible for the enhancement of oscillatory convection. Further it is important to not that the critical value of Pr_D(i.e, Pr_D^*), at which the system is most unstable decreases with both Ra_S and Le. These figures also reveal that the effect of Darcy–Prandtl number for $Pr_D > 10$ is less significant for the smaller values of Ra_S and Le.

The effect of Lewis number Le on the stability of the system in both oscillatory and finite amplitude modes for the different combination of parameters is depicted in Fig. 10(a–c). From these figures it is observed that the Le reduces the critical Rayleigh number for both the modes. This decreasing effect is more significant for small values of Le however for the moderate and large values of Le the stability thresholds are almost independent of Le. It is also observed that at each point the critical Rayleigh number for finite amplitude mode is less than that for the oscillatory mode. Thus the finite amplitude convection predominant over the oscillatory and stationary onsets.

In Fig. 10(a) the effect of mechanical anisotropy parameter ξ on the stability of the system is depicted for both oscillatory and finite

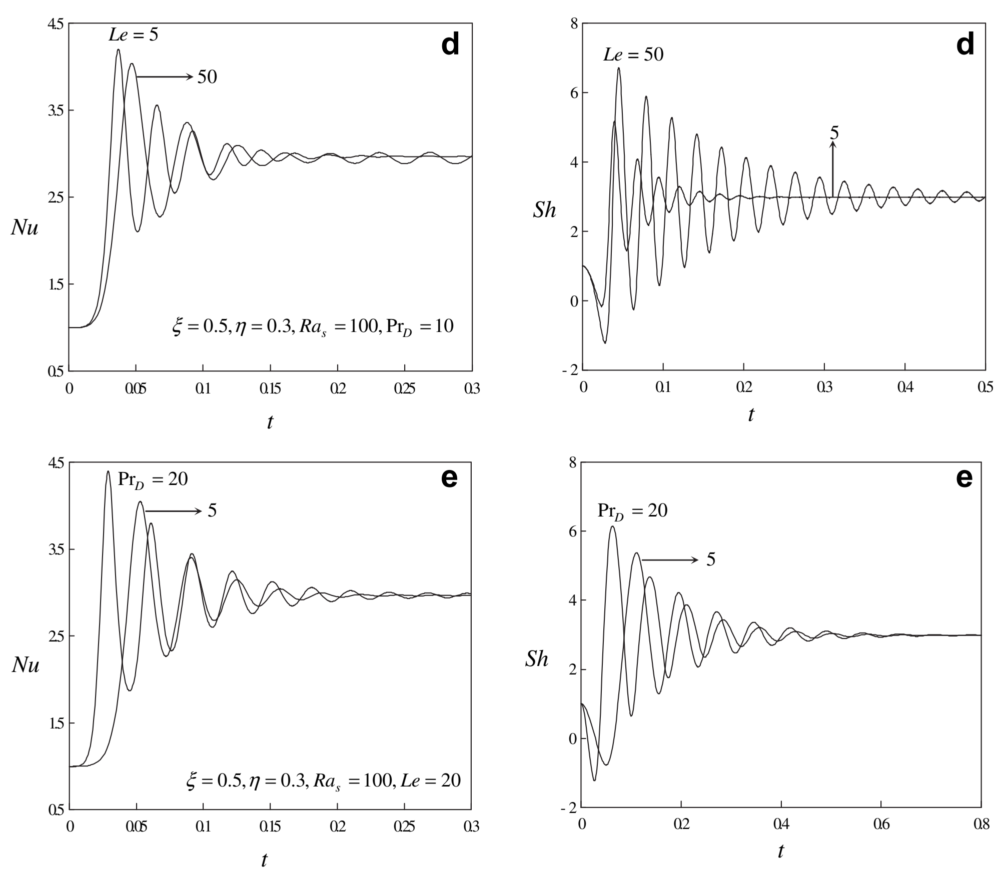

Fig. 12. (*continued*).

amplitude modes. It is observed that the effect of ξ is to destabilize the system in both the modes. The effect of thermal anisotropy parameter η on the oscillatory and finite amplitude critical Rayleigh number is shown in Fig. 10(b). In both modes the effect of η is to stabilize the system. The Rayleigh number for oscillatory mode become independent of η for very small values of Le, however in finite amplitude mode its effect is felt for all range of values of Le. Fig. 10(c) displays the effect of solute Rayleigh number on the onset criteria. The effect of Ra_S is to stabilize the system in both oscillatory and finite amplitude modes. It is important to note that the oscillatory and finite amplitude critical curves for $Ra_S = 0$ coincide with that of the stationary mode.

The weak non-linear analysis provides not only the onset threshold of finite amplitude motions but also the information of heat and mass transports in terms of Nusselt number and Sherwood number. The Nu and Sh are computed as the functions of Ra_T and the variation of these non-dimensional numbers with Ra_T for different parameters values is depicted in Fig. 11(a–d). It is observed that both Nu and Sh start from their conduction state value (i.e., unity) at the of onset of convection and increases with Ra_T. It is worth mentioning that near the point of onset the heat and mass transports are much sensitive to the temperature gradient. However when the Rayleigh number is increased beyond $2 \times Ra_{Tc}^{St}$ both Nu and Sh tends to a limiting value. It is also observed that the at every point the Sherwood number is greater than the Nusselt number. Therefore the mass transport is more substantial than the heat transport.

From Fig. 11(a and c) it is observed that the effect of mechanical anisotropy parameter and solute Rayleigh number is to enhance the heat and mass transports. In Fig. 11(b and d) the effect of thermal anisotropy parameter and Lewis number is displayed. It is found that with the increase of these parameters the heat transport is suppressed while the mass transport is reinforced.

The autonomous system of ordinary differential equations (4.7)–(4.11) is solved numerically using Runge–Kutta method. The values of unsteady finite amplitudes so obtained are then used to compute the Nusselt number and Sherwood number as function of time t. The variation of Nu and Sh with time t is presented though Fig. 12(a–e). It is observed that both Nu and Sh vary periodically with time. These periodic variations are of very short life. After a very short time is elapsed both Nu and Sh become steady. The transient behavior of the Nusselt number and Sherwood number for different values of ξ, η, Ra_S, Le and Pr_D is revealed respectively in Fig. 12(a–e).

6. Conclusions

The double diffusive convection in a horizontal anisotropic porous layer saturated with a Boussinesq fluid, which is heated and salted from below is studied analytically using both linear and non-linear stability theories. The effect of increasing the mechanical anisotropy parameter ξ is to advance the onset of oscillatory and finite amplitude convection. It is found that the thermal anisotropy parameter delay the onset of both oscillatory and finite amplitude convection. There exists critical value for solute Rayleigh number and Lewis number such that the convection mode switches from stationary to oscillatory when the values of these parameters exceeds the critical values. The Darcy–Prandtl number has a dual effect on the oscillatory mode in the sense that there is a critical value say Pr_D^* such that a strong destabilizing effect is observed when $Pr_D < Pr_D^*$, while for $Pr_D > Pr_D^*$ this effect is reversed and hence the system is stabilized. The stabilizing effect of Pr_D becomes less significant when $Pr_D >> Pr_D^*$. The Lewis number advances the onset of both oscillatory and finite amplitude convection while it delays the onset of stationary convection. In the unsteady case the transient behavior of the solute and thermal Nusselt numbers has been investigated, by solving numerically a fifth order Lorenz like model using Runge–Kutta method. Some of the convection systems previously reported in the literature is shown to be special cases of the system presented in this study.

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